Dos soluciones al desafío matemático nº 39, utilizando números complejos, publicadas en dos libros de resolución de problemas. Bruno Salgueiro Fanego, Viveiro, Lugo:

Del libro *Complex Numbers from A to… Z*, de Titu Andreescu y Dorin Andrica, editorial Birkhäuser, año 2006, capítulo 5 (Olympiad caliber problems), apartado 5.1 (Solving geometric problems), página 197:

Problem 10. *On the sides AB and AD of the triangle ABD draw externally squares $ABEF$ and $ADGH$ with centers O and Q, respectively. If M is the midpoint of the side BD, prove that OMQ is an isosceles triangle with a right angle at M.*

Solution. Let a, b, d be the coordinates of the points A, B, D, respectively.

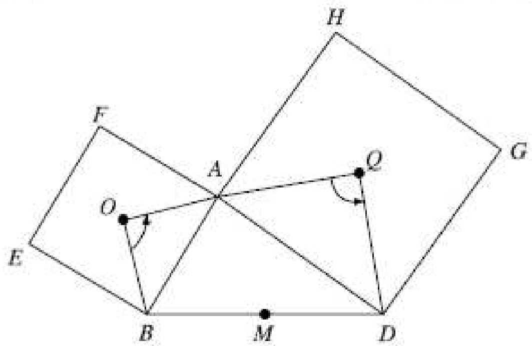

Figure 5.6.

The rotation formula gives

$$\frac{a - z_O}{b - z_O} = \frac{d - z_Q}{a - z_Q} = i,$$

so

$$z_O = \frac{b + a + (a - b)i}{2} \text{ and } z_Q = \frac{a + d + (d - a)i}{2}.$$

The coordinate of the midpoint M of segment $[BD]$ is $z_M = \dfrac{b + d}{2}$, hence

$$\frac{z_O - z_M}{z_Q - z_M} = \frac{a - d + (a - b)i}{a - b + (d - a)i} = i.$$

Therefore $QM \perp OM$ and $OM = QM$, as desired.

Del libro *Problem-solving strategies*, de Arthur Engel, editorial Springer-Verlag, año 1998, capítulo 12 (Geometry), apartado 12.1 (Vectors), páginas 296 y 297, problema más general que el desafío 39, y que lo califica de problema rutinario:

E12. *Squares $cbqp$ and $acmn$ are erected outwardly on the sides bc and ac of the triangle abc. Show that the midpoints d, e of these squares, the midpoint g of ab, and the midpoint f of mp are vertices of a square.*

This is a routine problem. Indeed, $gefd$ is a parallelogram since its vertices are midpoints of the sides of the quadrilateral $abpm$. We have just to show that eg and gd are perpendicular and of equal length. Indeed

$$g = \frac{a + b}{2}, \quad d = \frac{b + c}{2} + i\frac{b - c}{2}, \quad e = \frac{a + c}{2} + i\frac{c - a}{2},$$

$$d - g = \frac{c - a}{2} + i\frac{b - c}{2}, \quad e - g = \frac{c - b}{2} + i\frac{c - a}{2},$$

$$(d - g)i = \frac{c - b}{2} + i\frac{c - a}{2} = e - g.$$